Relative Motion Problems

$${}_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

match

SP1

${}_{p}\vec{V}_{a} = 200\,km/h$ [??]
airspeed ↖ heading

${}_{a}\vec{V}_{g} = 50\,km/h$ [E]

${}_{p}\vec{V}_{g} = ??$

a) heading [E]

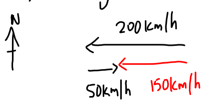
$${}_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

plane, wind

${}_{p}\vec{V}_{g} = 200\,km/h$ [E] $+ 50\,km/h$ [E]

${}_{p}\vec{V}_{g} = 250\,km/h$ [E]

b) heading [W]



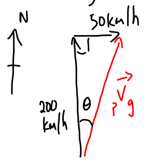
$${}_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

${}_{p}\vec{V}_{g} = 200\,km/h$ [W] $+ 50\,km/h$ [E]

${}_{p}\vec{V}_{g} = 200\,km/h$ [W] $- 50\,km/h$ [W]

${}_{p}\vec{V}_{g} = 150\,km/h$ [W]

c) heading [N]



$${}_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

${}_{p}\vec{V}_{g} = 200\,km/h$ [N] $+ 50\,km/h$ [E]

need to solve using the vector addition diagram.

$c^2 = a^2 + b^2$

$c^2 = 200^2 + 50^2$

$c = 206\,km/h$

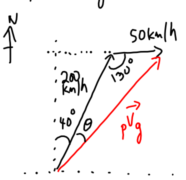
$\tan\theta = \frac{50}{200}$

$\theta = \tan^{-1}\left(\frac{50}{200}\right)$

$\theta = 14°$

${}_{p}\vec{V}_{g} = 206\,km/h$ [N 14°E]

d) heading [N40°E]



$${}_{p}\vec{V}_{g} = {}_{p}\vec{V}_{a} + {}_{a}\vec{V}_{g}$$

${}_{p}\vec{V}_{g} = 200\,km/h$ [N40°E] $+ 50\,km/h$ [E]

you must use a vector addition diagram

Law of Cosines:

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = 200^2 + 50^2 - 2(200)(50)\cos 130°$

$c = 235\,km/h$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B}$

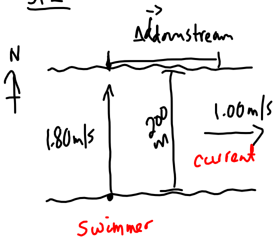
$\frac{50}{\sin\theta} = \frac{235}{\sin 130°}$

$\sin\theta = \frac{(50)\sin 130°}{(235)}$

$\theta = 9.4°$

${}_{p}\vec{V}_{g} = 235\,km/h$ [N49°E]

SP2



$_s\vec{V}_w = 1.80\text{ m/s }[N]$ ← swimmer

$_w\vec{V}_g = 1.00\text{ m/s }[E]$ ← current.

a) $_s\vec{V}_g = ?$

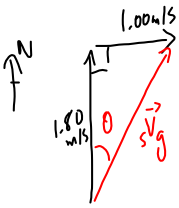
b) $\Delta t = ?$

c) $\Delta\vec{d}_{downstream} = ?$

$$_s\vec{V}_g = {_s\vec{V}_w} + {_w\vec{V}_g}$$

$$_s\vec{V}_g = 1.80\text{ m/s }[N] + 1.00\text{ m/s }[E]$$

must use a vector addition diagram



$c^2 = a^2 + b^2$

$c^2 = (1.00)^2 + (1.80)^2$

$c = 2.06\text{ m/s}$

a) $_s\vec{V}_g = 2.06\text{ m/s }[N\,29.1°E]$

$\tan\theta = \frac{1.00}{1.80}$

$\theta = \tan^{-1}\left(\frac{1.00}{1.80}\right)$

$\theta = 29.1°$

b) time to cross the river:

$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$

$\Delta t = \frac{\Delta\vec{d}}{\vec{V}}$

$\Delta t = \frac{200\text{ m }[N]}{1.80\text{ m/s }[N]}$

$\Delta t = 111\text{ s}$

* directions of $\Delta\vec{d}$ and $\vec{V}$ MUST match.

As long as you are headed perpendicular to the current/wind direction, the time to cross the river is the same as if there were no current.

c) $\Delta\vec{d}_{downstream} = ?$

↑ need the velocity of the current!

$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$

$\Delta\vec{d}_{downstream} = \vec{V}_{downstream}\Delta t$

$\Delta\vec{d}_{downstream} = (1.00\text{ m/s }[E])(111\text{ s})$

$\Delta\vec{d}_{downstream} = 111\text{ m }[E]$

In what direction would you head in order to go straight across?

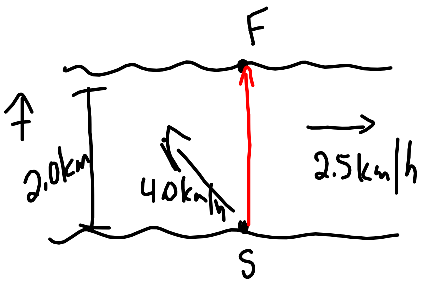
$\sin\theta = \frac{1.00}{1.80}$

$\theta = \sin^{-1}\left(\frac{1.00}{1.80}\right)$

$\theta = 33.7°$

head [N 33.7° W]

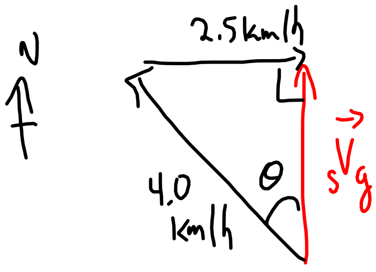
SP3



$_s\vec{v}_g$ = ___?___ [N]

$_s\vec{v}_w$ = 4.0km/h [??]

$_w\vec{v}_g$ = 2.5 km/h [E]



a) $\sin\theta = \frac{2.5}{4.0}$

$\theta = \sin^{-1}\left(\frac{2.5}{4.0}\right)$

$\theta = 39°$

The swimmer must head [N39°W]

b) time to cross: need to know the velocity across ($_s\vec{v}_g$)

$c^2 = a^2 + b^2$

$4.0^2 = 2.5^2 + b^2$

$b^2 = 4.0^2 - 2.5^2$

$b = 3.1$ km/h

$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$

$\Delta t = \frac{\Delta\vec{d}}{\vec{v}}$

$\Delta t = \frac{2.0\text{km [N]}}{3.1\text{km/h [N]}}$ > directions must match

$\Delta t = 0.64$ h